EXPERIMENTAL STUDY OF THE ATTENUATION OF PRESSURE WAVES IN FLOWS OF WEAK POLYACRYLAMIDE SOLUTIONS

V. G. Ivannikov and G. D. Rozenberg

UDC 532.513.1

The attenuation factor for a pressure wave generated by water hammer in the flow of a weak polyacrylamide solution is determined experimentally.

The unsteady flow equations for a viscoelastic fluid in an elastic pipe with a nonuniform velocity distribution in its cross section and a flow velocity much less than the velocity of sound have the form [1]

$$-\frac{\partial p}{\partial x}=\rho\left(\frac{\partial w}{\partial t}+\frac{\lambda|w|}{8\delta}w\right),\qquad -\frac{\partial p}{\partial t}=\rho c^2\frac{\partial w}{\partial x}. \tag{1}$$

It can be shown that expression (1) is also valid for the flow of non-Newtonian fluids, polymer solutions in particular, if the equation of state has the form [2]

$$\frac{d\rho}{\rho}=\frac{dp}{K},$$

where K = const is the bulk elastic modulus.

The presence of the dissipative term $(\lambda|w|/8\delta)w$ results in attenuation of the pressure (velocity) disturbances produced at the ends of the pipe. We have for the head values of the pressure waves in this case [3]

$$\bar{p}=\bar{p}_1\exp(-kx), \tag{2}$$

where $\bar{p}$ is the pressure disturbance above the steady value, $\bar{p}_1$ is the pressure disturbance in the cross section x = 0, and k is the attenuation factor.

Inasmuch as the dependence of λ on the flow parameters in the turbulent unstready regime is not known, k must be determined experimentally.

In the case of a pure water flow k can be fairly accurately determined from the expression [4]

$$k=\frac{\lambda_0|w_0|}{8c\delta}, \tag{3}$$

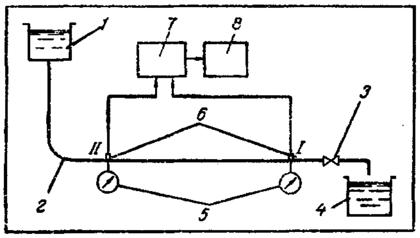

Fig. 1. Schematic of experimental arrangement.

where w_0 is the steady value of the velocity and λ_0 is the viscous friction coefficient for $w = w_0$.

We have undertaken an experimental study of the attenuation of a pressure shock in pipes for flows of weak solutions of polyacrylamide (PAA, processed according to Special Technical Specifications No. 0221-64) to determine the attenuation factor and its dependence on the solution concentration, to test the validity of Eq. (3), and to ascertain the influence of solution degradation on the value of the attenuation.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 25, No. 6, pp. 1045-1049, December, 1973. Original article submitted August 6, 1973.

TABLE 1. Experimental Attenuation Factor k_e

Fluid	w_0, m/sec	Number of tests	$k_e \cdot 10^4$, m^{-1}	$k \cdot 10^4$, m^{-1}	k_e/k
Water	0,69	21	3,20(1 ± 0,03)	2,96	1,08
	1,04	34	3,73(1 ± 0,05)	3,50	1,06
	1,30	25	5,15(1 ± 0,02)	4,75	1,08
	1,38	20	4,81(1 ± 0,01)	5,14	0,94
	1,62	30	5,75(1 ± 0,01)	5,81	0,99
PAA $C = 3 \cdot 10^{-4}$ g/cm³	1,57	23	5,28(1 ± 0,02)	2,24	2,36
	1,51	23	5,05(1 ± 0,02)	2,58	1,96
	1,57	30	5,53(1 ± 0,03)	2,03	2,72
	1,56	25	5,28(1 ± 0,03)	1,87	2,82
PAA $C = 2 \cdot 10^{-4}$ g/cm³	1,60	26	5,39(1 ± 0,03)	2,24	2,41
PAA $C = 1 \cdot 10^{-4}$ g/cm³	1,54	28	4,29(1 ± 0,03)	2,40	1,79
	1,53	29	4,68(1 ± 0,02)	2,57	1,82
PAA $C = 5 \cdot 10^{-5}$ g/cm³	1,58	25	4,98(1 ± 0,04)	2,68	1,86
	1,54	30	4,83(1 ± 0,03)	2,85	1,69
	1,60	24	4,56(1 ± 0,03)	2,91	1,57
PAA $C = 25 \cdot 10^{-6}$ g/cm³	1,59	19	4,96(1 ± 0,004)	2,96	1,67
	1,58	25	5,27(1 ± 0,02)	2,93	1,80
PAA $C = 1 \cdot 10^{-5}$ g/cm³	1,53	23	5,11(1 ± 0,02)	3,50	1,46
PAA $C = 5 \cdot 10^{-6}$ g/cm³	1,63	16	5,28(1 ± 0,02)	4,45	1,19

The experimental apparatus (Fig. 1) consisted of a delivery tank 1, a steel gravity-flow pipe 2 with a diameter of 25 mm and length of 110 m, a fast-acting valve (actuation time $t_V = 5 \cdot 10^{-3}$ sec), and a measuring receptacle 4.

The pressure at the ends of the test section I-II of length l = 73.4 m was measured for steady flow by the standard manometers 5 (at small flow velocities, the pressure drop was measured using a mercury differential manometer), and for unsteady flow by the DD-10 induction sensors 6. Signals from the sensors were amplified in an ID-2I instrument 7 and recorded on the H-700 light-beam oscillogram 8. The liquid flow rate at steady flow was determined by volumetric means.

The apparatus was calibrated with water in both steady and unsteady flow regimes.

In the steady regimes we obtained in the range $500 \leq \mathrm{Re} \leq 4 \cdot 10^4$ a curve of $\lambda(\mathrm{Re})$ that fits the function 64/Re for laminar flow and coincides with the Blasius curve for turbulent flow (the error not exceeding 5%).

In the unsteady regimes we measured the velocity of sound and the maximum shock pressure. The velocity of sound deviated at most 0.5% from the calculated value, and the shock pressure deviated at most 2% from the value calculated according to the formula of N. E. Zhukovskii. The velocity of sound in the PAA solution flows at all concentrations was the same as the velocity of sound in pure water.

The experimental value k_e of the attenuation was determined according to (2):

$$k_e = \frac{1}{l} \cdot \ln \frac{\bar{p}_1}{\bar{p}_2},$$

where $\bar{p}_2$ is the pressure disturbance in the cross section $x = l$ (section II in Fig. 1).

All of the tests with flows of water as well as the PAA solutions were repeated a number (20 to 30) of times, so that the experimental results could be subjected to statistical processing. The processing revealed that the relation $\sigma = \eta\sqrt{\pi/2}$ holds in every case, where σ is the rms value and η is the arithmetic-mean value of the individual-measurement error, i.e., the error distribution must be normal.

The experimental attenuation values k_e for water and the PAA solutions are given in Table 1, along with the solution concentrations C, the steady flow velocity w_0, and the number of tests n at the given

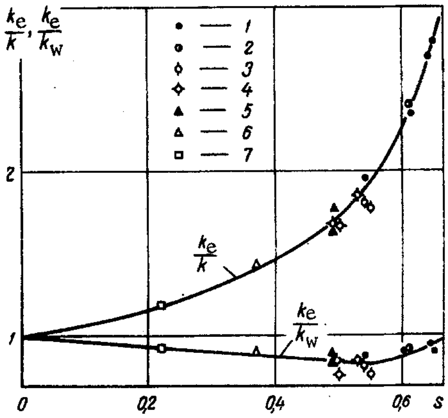

Fig. 2. Ratios k_e/k and k_e/k_W versus relative reduction s of the viscous friction for flows of PAA solutions at various concentrations: 1) $C = 3 \cdot 10^{-4}$; 2) $2 \cdot 10^{-4}$; 3) $1 \cdot 10^{-4}$; 4) $5 \cdot 10^{-5}$; 5) $25 \cdot 10^{-6}$; 6) $1 \cdot 10^{-5}$; 7) $5 \cdot 10^{-6}$ g/cm³.

velocity w_0. Also given for comparison are the attenuation values calculated according to Eq. (3). The values of λ_0 corresponding to the values of w_0 were determined experimentally.

The dependence of k_e/k and k_e/k_W on $s = 1 - \lambda_{op}/\lambda_{ow}$ is given in Fig. 2. Here λ_{op} and λ_{ow} are the viscous friction coefficients for steady flows of the PAA solutions and water at the respective velocities w_{op} and w_{ow}. The coefficients λ_{op} and λ_{ow} were determined experimentally under the condition $w_{ow} = w_{op}$.

It is evident from Table 1 and Fig. 2 that Eq. (3), which holds for water, is inapplicable to flows of weak PAA solutions; the deviation from Eq. (3) increases with the net effect of the polymer, i.e., as the value of s increases; the absolute value of the attenuation of the pressure head wave for weak PAA solutions is somewhat lower than for water and attains a minimum for $s \approx 0.5$.

Simultaneously with the experiments described above we also investigated the influence of degradation of the PAA solution on the attenuation. We performed four series of identical tests with a solution at $C = 3 \cdot 10^{-4}$ g/cm³.

It turned out that successive water hammers essentially do not produce degradation of the solution. In a series of 80 water hammers following one immediately after another we were unable to detect any regular variation of the attenuation.

Accelerated degradation of the solution took place when it was transferred from the measuring receptacle 4 into the delivery tank 1 by means of a high-speed centrifugal pump. After each transfer we measured the viscous friction coefficient λ_{op} and generated a series of water hammers.

The experimental results are summarized in Figs. 3a and 3b. It is evident from the graphs that as the polymer solution is degraded the disparity between the experimental value k_e of the attenuation and the attenuation k calculated according to Eq. (3) continuously diminishes.

We also note that the dependence of k_e/k_W on s, as in the case of the undegraded solutions (see Fig. 2), has a minimum at $s \approx 0.5$.

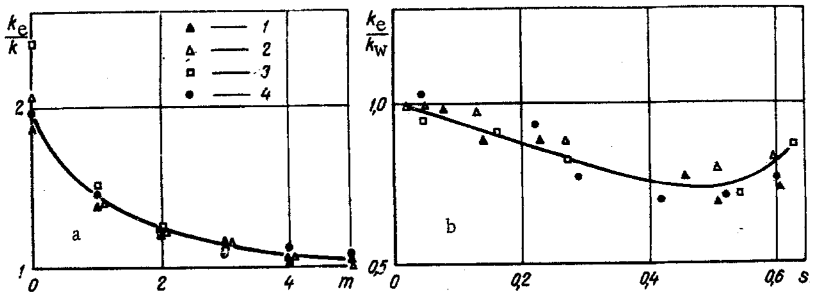

Fig. 3. Ratio k_e/k versus number m of pumpings of the total volume of PAA solution (a) and versus relative reduction s of the viscous friction with degradation of the PAA solution (b) for an initial concentration $C = 3 \cdot 10^{-4}$ g/cm³ in four series of identical tests (1-4).

NOTATION

p, ρ, w	are the pressure, density, and flow velocity of the fluid, averaged over pipe cross section;
w_0, w_{0p}, w_{0w}	are the steady flow velocities of the fluid, the polymer solution, and water;
c	is the reduced velocity of sound;
δ	is the hydraulic radius;
λ, λ_0	are the viscous friction coefficients for unsteady and steady flow;
λ_{0p}, λ_{0w}	are the viscous friction coefficients for steady flow of the polymer solution and water;
K	is the bulk elastic modulus;
l	is the measurement length in the pipe;
k	is the pressure-wave attenuation factor calculated according to (3);
k_e	is the experimental value of the attenuation factor;
k_w	is the attenuation factor calculated for water according to (3);
C	is the concentration of the PAA solution;
$s = (\lambda_{0w} - \lambda_{0p})/\lambda_{0w}$	is the relative reduction of the viscous friction;
σ	is the rms individual-measurement error;
η	is the arithmetic-mean individual-measurement error;
$\bar{p}$	is the pressure disturbance in excess of the steady value.

LITERATURE CITED

1. I. A. Charnyi, Unsteady Motion of a Real Fluid in Pipes [in Russian], GITTL, Moscow—Leningrad (1951).
2. G. D. Rozenberg, Izv. VUZ. Neft' i Gaz, No. 8 (1959).
3. I. A. Charnyi and G. D. Rozenberg, Appendix to the Russian translation of: L. J. B. Bergeron, Water Hammer in Hydraulics and Wave Surges in Electricity, Mashgiz, Moscow (1962) [English edition: Wiley, New York (1961)].
4. G. D. Rozenberg, Dokl. Akad. Nauk SSSR, 129, No. 1 (1959).